Algebra 1, Quarter 3, Unit 3.1

Continuing Quadratic/Polynomial Real-World Problems

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand closed operations.
- Add, subtract, and multiply polynomials.
- Create equations in two or more variables.
- Graph equations on coordinate axes with labels and scales.
- Represent constraints of equations or inequalities and systems of equations and/or inequalities.
- Interpret solutions as viable and nonviable with modeling.
- Solve quadratic equations using completing the square.
- Solve quadratic equations by taking square roots.
- Solve quadratic equations using the quadratic formula.
- Solve quadratic equations using factoring.
- Explain why the x coordinate of the point of intersection of the graphs of the equations $y = f(x)$ and $y = g(x)$ is the solution to the equation $f(x) = g(x)$.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use scales and units to construct graphs that appropriately represent data.
- Analyze constraints.
- Explain correspondences among equations, verbal descriptions, tables, and graphs; draw diagrams of important features and relationships. Graph data and search for regularity or trends.

Construct viable arguments and critique the reasoning of others.

- Reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- Compare the effectiveness of two plausible arguments. Distinguish correct logic or reasoning from that which is flawed.

Model with mathematics.

- Solve real-world mathematics problems using quadratic equations and inequalities.
- Identify important quantities in a practical situation and map their relationships using such tools as graphs and formulas.
- Analyze relationships mathematically to draw conclusions.

Essential questions

- What are the connections among the solutions (roots) of quadratic equations, the zeros of their related functions, and the horizontal intercepts of the graph of the function?
- What are the different ways to solve quadratic equations and when is each appropriate?
- How are quadratic equations used in the "real" world?
- How are adding and multiplying polynomials different?

Written Curriculum

Common Core State Standards for Mathematical Content

Arithmetic with Polynomials and Rational Expressions **A-APR**

Perform arithmetic operations on polynomials [~~*Linear and*~~ *quadratic*]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Creating Equations* **A-CED**

Create equations that describe numbers or relationships [~~*Linear,*~~ *quadratic,* ~~*and exponential (integer inputs only); for A.CED.3 linear only*~~]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from* ~~*linear and*~~ *quadratic functions,* ~~*and simple rational and exponential functions.*~~*

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.**

Reasoning with Equations and Inequalities **A-REI**

Solve equations and inequalities in one variable [~~*Linear inequalities; literal that are linear in the variables being solved for;*~~ *quadratics with real solutions*]

A-REI.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations [~~*Linear-linear and*~~ *linear-quadratic*]

A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ and the circle $x^2 + y^2 = 3$.

Represent and solve equations and inequalities graphically

A-REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are ~~linear,~~ polynomial~~, rational, absolute value, exponential, and logarithmic~~ functions.★

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In kindergarten students began representing addition and subtraction with equations. First and second grade students wrote equations with unknowns in all positions. In third grade, students began writing equations involving multiplication and division. In fourth grade, they used verbal statements and variables when working with equations. Sixth grade students set up and solved one-step equations and inequalities. Seventh grade students solved real-world problems involving expressions and equations. Eighth grade students solved linear equations and pairs of linear equations, and they also solved multi-step equations.

Current Learning

Students understand the closed operations of addition, subtraction, and multiplication (including order of operations). They add, subtract, and multiply polynomials. Students create equations and inequalities in one variable and use them to solve problems. They create equations in two or more variables to represent relationships between quantities. Students graph equations on the coordinate axes using labels and scales. Students solve quadratic equations in one variable by completing the square, taking square roots, using the quadratic formula, and factoring. They represent and solve equations and inequalities graphically.

Future Learning

In algebra 2, students will create equations that describe numbers or relationships. They will also perform arithmetic operations on polynomials beyond quadratics. Careers using these skills include insurance underwriter, economist, various engineering fields, nursing and medical fields, payroll personnel, physicist, astronomers, and computer software engineers.

Additional Findings

According to *PARCC Model Content Frameworks—Mathematics, Grades 3–11*, Version 2.0, August 31, 2012 (pp. 70–78):

- A-CED.1 – Tasks are limited to linear or exponential equations with integer exponents.
- A-REI.4b – Task do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. Students recognize cases in which a quadratic equation has no real solutions.
- A-REI.11 – Does not include exponents and logarithmic functions. Find solutions approximately for polynomial functions only.

Algebra 1, Quarter 3, Unit 3.2

Representing Polynomial Functions

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Understand the concept of a function.
- Use function notation.
- Evaluate functions for inputs in their domains.
- For functions that model a relationship between two quantities:
  - Interpret key features of graphs.
  - Sketch graphs.
  - Show key features given a verbal description of the relationship.
- Graph functions expressed symbolically by hand and using technology in more complex cases.
- Write a function defined by an expression in different but equivalent forms.
- Compare properties of two functions each represented in a different way.

Mathematical practices to be integrated

Model with mathematics.

- Use scales and units to make graphs that appropriately represent data.
- Make conjectures about the form and meaning of solutions.
- Search for regularity or trends.

Use appropriate tools strategically.

- Use graphing calculators.
- Use rulers, pencil, paper, and concrete models.

Attend to precision.

- Take care in specifying units of measure and labeling axes to clarify the correspondence of quantities in a problem.
- Express numerical answers with a degree of precision appropriate for the problem context.

Essential questions

- How can you use functions to model real-world situations?
- What are the characteristics of quadratic functions represented both graphically and algebraically?
- How can you compare functions that are represented in different ways?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions	**F-IF**

Understand the concept of a function and use function notation [*Learn as general principle; focus on ~~linear and~~ exponential and on arithmetic and geometric sequences*]

F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context [*~~Linear~~, ~~exponential,~~ and quadratic*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

Analyze functions using different representations [*~~Linear~~, exponential, quadratic, ~~absolute value, step, piecewise-defined~~*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions **F-BF**

Build a function that models a relationship between two quantities [*For F.BF.1, 2, ~~linear,~~ exponential, and quadratic*]

F-BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

Build new functions from existing functions [*Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only*]

F-BF.3 Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and

compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have.

Clarifying the Standards

Prior Learning

In kindergarten students began representing addition and subtraction with equations. First and second grade students wrote equations with unknowns in all positions. In third grade, students began writing equations involving multiplication and division. In fourth grade, they used verbal statements and variables when working with equations. Sixth grade students set up and solved one-step equations and inequalities. Seventh grade students solved real-world problems involving expressions and equations. Eighth grade students solved linear equations and pairs of linear equations, and they also solved multi-step equations.

Current Learning

Students understand the concept of a function and use function notation. For functions that model a relationship between two quantities, they sketch graphs showing key features and give a verbal description of the relationship. Students relate the domain of a function to its graph. They graph functions expressed symbolically and show key features of the graph (intercepts, maxima, and minima). Students write functions in different, but equivalent forms. They use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and they interpret these in terms of a context. Students compare two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description). They write a function that describes a relationship between two quantities. Students determine an explicit expression, a recursive process, or steps for calculation from a context. They identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, and $f(x+k)$ for specific values of k (both positive and negative), and they find the value of k given the graphs.

Future Learning

In algebra 2, given functions that model a relationship between two quantities, students interpret key features of graphs and tables in terms of the quantities. They sketch graphs showing key features given a verbal description of the relationship (polynomial, exponential, logarithmic, and trigonometric functions). They also compare properties of two functions represented in a different ways. Careers using these skills include insurance underwriter, economist, various engineering fields, nursing and medical fields, payroll personnel, physicist, astronomers, and computer software engineers.

Additional Findings

According to *PARCC Model Content Frameworks–Mathematics, Grades 3–11*, Version 2.0, August 31, 2012 (pp. 70-78):

- F-IF.4 – Tasks have a real-world context. Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
- F-IF.9 – Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

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Algebra 1, Quarter 3, Unit 3.3

Interpreting the Structure of Expressions and Using Equations and Inequalities to Solve Problems

Overview

Number of instructional days: 8 (1 day = 45–60 minutes)

Content to be learned

- Interpret the structure of expressions, including terms, factors, and coefficients.
- Use the structure of an expression to identify ways to rewrite the expression.
- Create equations and inequalities in one variable.
- Use equation and inequalities to solve problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.

Attend to precision.

- Communicate precisely to others clear definitions and state the meanings of symbols they chose.

Essential questions

- How would you interpret the parts of a function?
- What are examples of how the structure of an expression can be used to rewrite the expression?
- How can equations and inequalities be used in real-world situations?

Written Curriculum

Common Core State Standards for Mathematical Content

The Real Number System **N-RN**

Extend the properties of exponents to rational exponents.

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

The Real Number System **N-RN**

Extend the properties of exponents to rational exponents.

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Seeing Structure in Expressions **A-SSE**

Interpret the structure of expressions [~~*Linear*~~*, exponential,* ~~*quadratic*~~]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Creating Equations★ **A-CED**

Create equations that describe numbers or relationships [~~*Linear, quadratic, and*~~ *exponential (integer inputs only); for A.CED.3 linear only*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear* ~~*and quadratic functions, and simple rational and*~~ *exponential functions.*★

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grades 6 and 7, students used exponents to find volume and area of a cube. In 8^{th} grade, they applied properties of integer exponents and used square roots and perfect cube roots. Students also used scientific notation in grade 8.

Current Learning

Students interpret expressions that represent a quantity in terms of its context (e.g., terms, factors, and coefficients). They interpret complicated expressions and identify ways to rewrite expressions. Students create equations and inequalities and use them to solve problems.

Future Learning

In algebra 2, students will use the structure of an expression to identify ways to rewrite it (polynomial, rational, or exponential expressions). The following are examples that students will encounter:

- Understanding $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
- Seeing the equation $x^2 + 2x + 1 + y^2 = 9$ as an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (-1, 0).
- Seeing $(x^2 + 4)/(x^2 + 3)$ as $[(x^2+3) + 1]/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.

When creating equations and inequalities in one variable and using them to solve problems, tasks will be limited to exponential equations with rational or real exponents and will have a real-world context.

Careers using these skills include insurance underwriter, economist, various engineering fields, nursing and medical fields, payroll personnel, physicist, astronomers, and computer software engineers.

Additional Findings

According to PARCC Model Content Frameworks – Mathematics, Grades 3–11, Version 2.0, August 31, 2012 (pp. 70-78):

- A-CED.1 – Tasks are limited to linear or exponential equations with integer exponents.
- A-SSE.1 Tasks are limited to numerical expressions and polynomial expressions in one variable. Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form (53+47)(53-47). See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.

Algebra 1, Quarter 3, Unit 3.4

Interpreting and Analyzing Functions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Interpret key features of graphs and tables in terms of quantities for a function that models a relationship between quantities.
- Sketch graphs of functions showing key features, given a verbal description of the functional relationship.
- Calculate and interpret the average rate of change of a function.
- Compare properties of two functions each represented in different ways.

Mathematical practices to be integrated

Model with mathematics.

- Identify important quantities in a practical situation and map their relationships using such tools as graphs and formulas.

Use appropriate tools strategically.

- Use graphing calculators.
- Use rulers, pencil, paper, and concrete models.

Attend to precision.

- Take care in specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem.
- Express numerical answers with a degree of precision appropriate for the problem context.

Essential questions

- How can the rate of change be found in various representations of data?
- How does a domain relate to a function?
- How do the attributes of algebraic and graphic functions compare?

Written Curriculum

Common Core State Standards for Mathematical Content

Interpreting Functions F-IF

Interpret functions that arise in applications in terms of the context [~~*Linear*~~, ~~*exponential, and*~~ *quadratic*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations [*Linear,* ~~*exponential*~~*, quadratic,* ~~*absolute value, step, piecewise-defined*~~]

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In kindergarten, students described and compared data. In grades 1–5, students represented and interpreted data. Second grade students graphed picture and bar graphs. In third grade, students solved one- and two-step problems using information presented in a scaled bar graph. Fifth grade students graphed ordered pairs on a coordinate plane. In sixth grade, they represented and analyzed quantitative relationships between dependent and independent variables. Students continued graphing linear equations in grade 7. In eighth grade, students analyzed and solved linear equations and pairs of simultaneous linear equations. They used functions to model relationships between quantities. Students also learned that a function is a rule that assigns each input to exactly one output. They learned how to graph a function using a set of ordered pairs, and they learned that functions can be represented in different ways, including graphically, numerically in tables, or by verbal description. Students also learned how to interpret a slope–intercept form equation ($y = mx + b$) and that this type of equation represents a linear function. They also learned to determine when a function is not linear.

Current Learning

Students interpret key features of graphs and tables in terms of quantities for a quadratic function that models a relationship between quantities. They sketch a graph showing key features given a verbal description of a quadratic function relationship. Students relate the domain of a quadratic function to its graphs, and they calculate and interpret the average rate of change of a function. They compare properties of two functions each represented in different ways.

Future Learning

For a function that models a relationship between two quantities, Algebra 2 students will interpret key features of graphs and tables in terms of quantities and will sketch graphs showing key features given a verbal description of the relationships (real-world context). Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

Careers using these skills include insurance underwriter, economist, various engineering fields, nursing and medical fields, payroll personnel, physicists, astronomers, and computer software engineers.

Additional Findings

According to *PARCC Model Content Frameworks – Mathematics, Grades 3–11*, Version 2.0, August 31, 2012 (pp. 70-78):

- F-IF.4 – tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
- F-IF.6 – tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.